Old and New Algorithms for Blind Deconvolution

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joint work with Anat Levin, WIS

(thanks to Meir Feder, EE, TAU)
Blind Deblurring

\[ y = x \ast k \]
Much Recent Progress

(Fergus et al. 06, Cho et al. 07, Jia 07, Joshi et al. 08, Whyte et al. 10, Shan et al. 07, Harmeling et al. 10, Sroubek and Milanfar 11 · · · )
MAP using sparse derivative prior

\[
\log P(x, k|y) = C + \sum_i |x_i|^\alpha + \lambda \|k * x - y\|^2
\]

- \text{MAP}_{xk}: (x^*, k^*) = \arg \max P(x, k|y). \text{ Guaranteed to fail with global optimization. Often works well in practice.}
- \text{MAP}_k: (k^*) = \arg \max_k \int_x P(x, k|y). \text{ Guaranteed to succeed if images sampled from prior. Can be difficult to optimize.}

(Levin et al. 09)
This talk:

- Old algorithms for blind deconvolution from communication systems. ≠ MAP. Rigorous correctness proofs.
- Can explain success of some $MAP_{x,k}$ algorithms.
**Blind Deconvolution in Communication**

- \( x(t) \) the transmitted signal.
- \( y(t) = \sum_k k \tau x(t - \tau) \) received signal.
- \( y = x \ast k \)

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**Self-Recovering Equalization and Carrier Tracking in Two-Dimensional Data Communication Systems**

DOMINIQUE N. GODARD, MEMBER, IEEE

Abstract—Conventional equalization and carrier recovery algorithms for minimizing mean-square error in digital communication systems generally require an initial training period during which a known data sequence is transmitted and properly synchronized at the receiver. This paper solves the general problem of adaptive channel equalization without resuming to a known training sequence or to conditions of limited distortion. The criterion for equalizer adaptation is the minimization of a new class of nonconvex cost functions which are shown to characterize intersymbol interference independently of carrier phasing and of the data symbol constellation used in the transmission system. Equalizer convergence does not require carrier recovery, so that carrier phase tracking can be carried out at the equalizer output in a decision-directed mode. The convergence properties of the self-recovering algorithms are analyzed mathematically and confirmed by computer simulation.

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**New Criteria for Blind Deconvolution of Nonminimum Phase Systems (Channels)**

OFIR SHALVI AND EHUD WEINSTEIN, SENIOR MEMBER, IEEE

Abstract—A necessary and sufficient condition is derived for blind deconvolution (without observing the input) of nonminimum phase linear time-invariant systems (channels). Based on this condition, several optimization criteria are proposed, and their solution is shown to correspond to the desired response. These criteria only involve the computation of second- and fourth-order moments, implying a simple tap update procedure. The proposed methods are universal in the sense that they do not impose any restrictions on the probability distribution of the (unobserved) input sequence. We also address the problem of additive noise in the system and show that in several important cases (e.g., when the additive noise is Gaussian) the proposed criteria are essentially unaffected.

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Beveniste et al. [4] present several concepts and results that significantly contribute to the understanding of the problem. First, they establish that a criterion based on second-order statistics, e.g., the mse criterion, is insufficient for phase identification. For that reason the prob-
Blur makes signals Gaussians (Central Limit Theorem)

Orig. Signal (IID)  
Blurred

Histogram ($\kappa=1$)

$$\kappa(y) = \frac{1}{N} \sum_i \bar{y}_i^4, \bar{y}_i = y_i/\text{std}(y)$$  
(Shalvi and Weinstein 1990)
Blur makes signals Gaussians (Central Limit Theorem)

Orig. Signal (IID)  Histogram ($\kappa=26.43$)

Blurred  Histogram ($\kappa=5.45$)
Simple Blind Deconvolution Algorithm (Shalvi and Weinstein 1980)

Assume $x_i$ IID. \( y = x \ast k \). Solve for the “inverse filter” \( e \) such that \( e \ast y = x \).

- \( x_i \) sub-Gaussian (\( \kappa < 3 \)):
  \[
  e^* = \arg \min_e \kappa(e \ast y)
  \]
  (Godard 1970)

- \( x_i \) super-Gaussian (\( \kappa > 3 \)):
  \[
  e^* = \arg \max_e \kappa(e \ast y)
  \]

Claim: Guaranteed to find the correct \( e \): \( e^* \ast y = x \). No local minima.
Simple Proof

Assume $x_i$ IID. $x_i$ super-Gaussian ($\kappa > 3$): $y = x \ast k$.

$$e^* = \arg\max_e \kappa(e \ast y)$$

**Proof:** $\hat{x} = e \ast y = e \ast k \ast x = (e \ast k) \ast x$

$\Rightarrow$ Unless $e \ast k = \delta$, $\hat{x}$ is more Gaussian than $x$ so $\kappa(\hat{x}) < \kappa(x)$
Proof by pictures:

Orig. Signal (IID) (\(\kappa=26.43\))

Blurred (\(\kappa=5.45\))

deblurred (wrong \(e\)) (\(\kappa=14.14\))
Assume $x_i$ IID. $x_i$ super-Gaussian. ($\kappa > 3$): $y = x \ast k$.

$$e^* = \arg \max_e \kappa(e \ast y)$$

- Universal proof of correctness (don’t need to know $\Pr(x_i)$, just IID and sub/super Gaussianity).
- Proofs of global convergence of iterative algorithms.
- Used in microwave radio transmissions (1980s), cable set-top boxes (mid 1990s) and wireless communication (late 1990s-today) (Johnson et al. 1998)
- Can we use this for blind image deblurring?
Blur makes derivatives more Gaussian

- Sharp image dx
- Blurred image dx

Histogram \((\kappa = 20.02)\)

Histogram \((\kappa = 17.99)\)
New class of image blind deblurring algorithms

\( y \): blurred (deriv) image  \quad \text{Histogram (}\kappa = 17.99\text{)}

\( x \): sharp (deriv) image  \quad \text{Histogram (}\kappa = 20.02\text{)}

Find \((x^*, k^*)\) such that:

- \( y = x^* * k^* \)
- \( x^* \) as non Gaussian as possible.

If derivatives are IID, this algorithm provably finds the correct blur kernel.
Measuring non Gaussianity using normalized moments

\[ \frac{1}{N} \sum_i \bar{x}_i^\alpha, \quad \bar{x}_i = x_i / \text{std}(x) \]

\begin{align*}
\sum_i \bar{x}_i^4 &= 25.73 \\
\sum_i \bar{x}_i^{1/2} &= 0.60 \\
\sum_i \bar{x}_i^4 &= 5.8 \\
\sum_i \bar{x}_i^{1/2} &= 0.74 \\
\sum_i \bar{x}_i^4 &= 3 \\
\sum_i \bar{x}_i^{1/2} &= 0.81
\end{align*}
A new(?) algorithm

\( y: \) Blurred image \hspace{1cm} \text{Histogram (\( \kappa = 17.99 \))}

\[
(x^*, k^*) = \arg \min_{x, k} \sum_i |\tilde{x}_i|^{\alpha} + \lambda \| k \ast x - y \|^2
\]

\( \tilde{x}_i = x_i / \text{std}(x) \)

Guaranteed to succeed.
MAP_{x,k} algorithm

\( y: \) Blurred image \hspace{1cm} \text{Histogram} (\kappa = 17.99)

\[
(x^*, k^*) = \arg \min_{x,k} \sum_i |x_i|^{\alpha} + \lambda \|k * x - y\|^2
\]

Guaranteed to fail.
Normalized Sparsity (Krishnan et al. 11)

\[
\min_{x,k} \lambda \| x \ast k - y \|^2 + \frac{\| x \|_1}{\| x \|_2} + \psi \| k \|_1
\]

Can be seen as special case of “new” algorithm.
\( \text{MAP}_{xk} \) with bilateral filtering (Cho and Lee 09, Hirsch et al. 11)

\[
\begin{align*}
\bar{x} & \leftarrow \text{BilateralFilter}(x) \\
k & \leftarrow \arg \min_k \|y - k \ast \bar{x}\| \\
x & \leftarrow \arg \min_x \lambda \|y - k \ast x\|^2 + \text{sparsity}(x)
\end{align*}
\]

Can be seen as approximating special case of “new” algorithm.
A new(?) algorithm

$y$: Blurred image

Histogram ($\kappa = 17.99$)

$$(x^*, k^*) = \arg \min_{x,k} \sum_i |\tilde{x}_i|^{\alpha} + \lambda \|k * x - y\|^2$$

$\tilde{x}_i = x_i / \text{std}(x)$

Guaranteed to succeed.
So what does this buy us?

- Understanding recent algorithms. Proof of success.
- Improving recent algorithms. Filters with better independence properties. Iterative algorithms with global convergence.
Conclusions

• MAP algorithms for blind image deconvolution. May work even when not supposed to.
• Old algorithms for blind deconvolution in communications. Universal guarantees, global convergence, used in millions of devices.
• Can help us understand and improve image deblurring algorithms.